Functional Analysis - Few Comparisons with Real Analysis and Some Applications

Kshitiz Mangal Bajracharya
Roll Number : 9
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Differences between Real Analysis and Functional Analysis

- *Elements of interest* : In Real Analysis, the elements of interest are **real numbers**. There, we generally study about real numbers, their properties and functions (usually real-valued) defined on them. In Functional Analysis, the elements of interest are **vectors**, which can be numbers, sequences, functions, etc. The properties of functions studied in Real Analysis is a base for Functional Analysis.

Differences between Real Analysis and Functional Analysis

- *Notion of Linearity* : In Real Analysis, $f : A \longrightarrow \mathbf{R}$ is said to be linear if $f(x) = ax + b$. But in Functional Analysis, $f : X \longrightarrow Y$ is said to be linear if $f(ax + by) = af(x) + bf(y)$. For example, $f : \mathbf{R} \longrightarrow \mathbf{R}$ defined by $f(x) = 2x + 3$ is linear in first sense but not in second sense.

Differences between Real Analysis and Functional Analysis

- *Notion of Boundedness* : In Real Analysis, $f : A \longrightarrow \mathbf{R}$ is said to be bounded if

$$\exists\ c > 0 :\ \forall\ x \in A |f(x)| \leq c.$$

But in Functional Analysis, $f : X \longrightarrow \mathbf{K}$ is said to be bounded if

$$\exists\ c > 0 :\ \forall\ x \in X |f(x)| \leq c\|x\|.$$

For example, $f : \mathbf{R} \longrightarrow \mathbf{R}$ defined by $f(x) = x$ is bounded in second sense but not in first sense.

Need of Functional Analysis in other areas of Mathematics

- *In Differential and Integral Equations* : Establishing fundamental results related to uniqueness theorems and method of undetermined coefficients. Also, used in establishing results related to Fredholm's and Volterra's Integral Equations.
- *In Special Functions, Harmonic Analysis and Fourier Analysis* : Establishing basic orthogonal polynomials due to Legendre, Hermite and Laguerre. C^*-algebras are used extensively in Harmonic Analysis. The famous Fourier Series can also be established using IPS.

Need of Functional Analysis in other areas of Mathematics

- *In Numerical Analysis* : In finite element method (spectral theory) and approximation theory (fixed points).

Apart from these, it is also applicable in other fields of study like computer science, quantum physics and engineering.